

Aspects of the Vertical Dynamics of a Car Suspension

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Studying the dynamics of a car suspension we soon face the two resonance points of the system. The first exists at approx. 1 Hz. Its parameters are the mass of the vehicle's body (chassis) and the adherent spring constant. The second lies at approx. 10 Hz. Its parameters are the wheel mass and the spring constant of the tyre. Both resonances are damped by the same shock absorber. That works because the aperiodic damping coefficient is the same for both resonance cases.

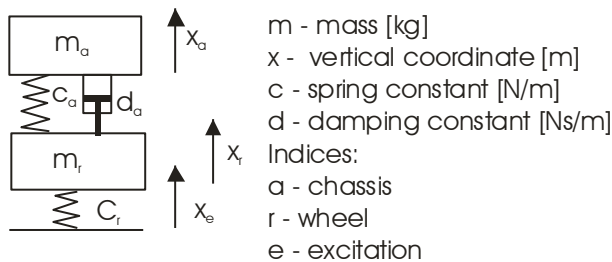


Fig.1 One wheel model (quarter car)

If the damper has got a tough adjustment the influence of the spring is reduced. The vehicle's body mass and the tyre spring then create a new resonance point at approx. 3 Hz which replaces the resonances of 1 Hz and 10 Hz. Its parameters are vehicle mass (chassis) and tyre spring constant. This oscillation decreases slowly because of the low damping coefficient of the tyre. All resonances are situated within the range of operation.

The diff. eq. following Newton are ($\dot{x} = dx/dt$):

$$\begin{aligned}
 m_a \ddot{x}_a &= c_a (x_r - x_a) + d_a (\dot{x}_r - \dot{x}_a) \\
 m_r \ddot{x}_r &= c_r (x_e - x_r) - c_a (x_r - x_a) - d_a (\dot{x}_r - \dot{x}_a) \\
 d_a &= 2 \cdot (m_a \cdot c_a)^{0.5} \cdot d_{rel}
 \end{aligned}$$

Box 1

The 1 Hz Resonance

According to Fig.1 one can calculate the transfer functions by using the differential equations of box 1 (differential equations without the force component of the weight). In Fig.2 they are presented for the vertical coordinates of chassis and wheel. The applied parameter values are as usual. The graphs show in the range of the 1 Hz resonance an amplitude hump of

the chassis coordinate and of the wheel coordinate, too, but the latter in a smaller scale.

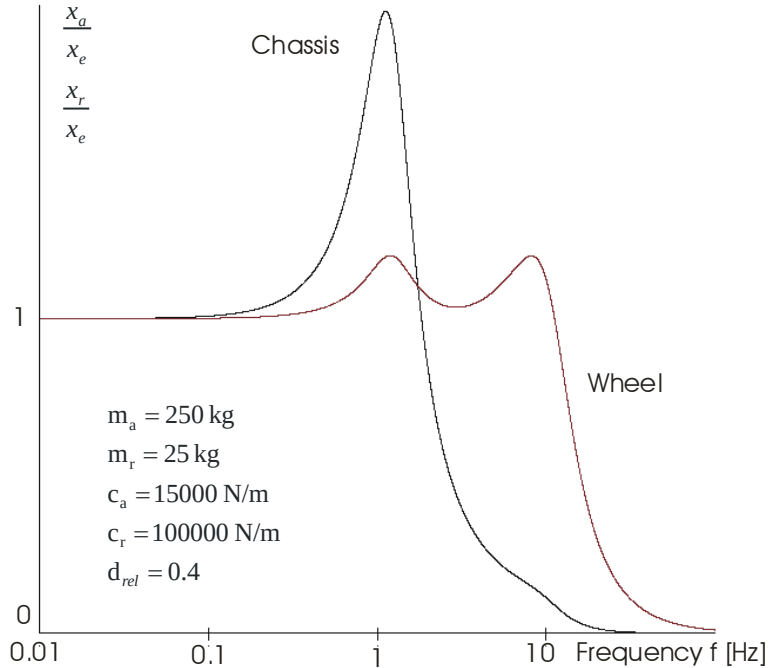


Fig. 2 Transfer Function (2 mass oscillator)

The resonance hump reduces the riding comfort and it would be desirable to make the slope of the transfer function flatter.

The desirable properties of the vertical dynamic are a good oscillation isolation in the range of $> 1 \text{ Hz}$ and level constancy (in relation to the road) in the frequency range of $< 1\text{-}2 \text{ Hz}$. The intermediate range of the transfer function should include a smooth transition between the amplitude ratio of 1 and of near to zero.

The safety is a boundary condition which should never be neglected. It is defined by a constant pressure of the tyre on the road.

Fig. 3 shows the transfer functions of chassis acceleration and wheel movements.

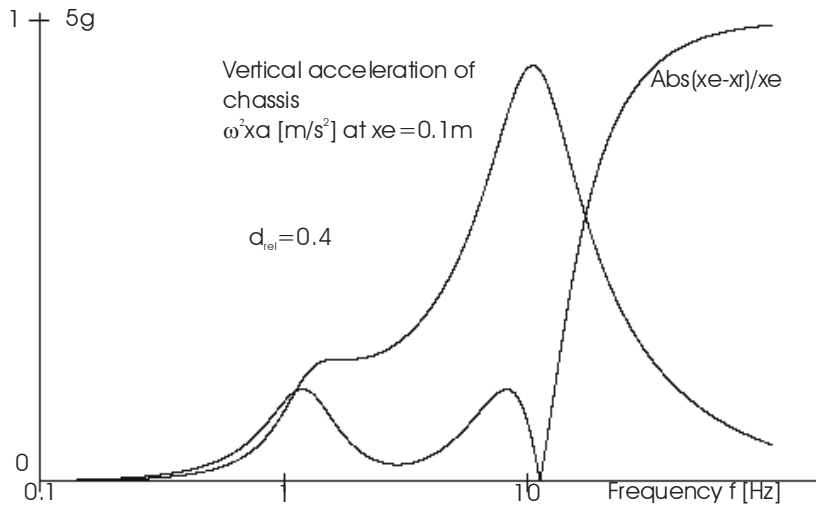


Fig. 3 Transfer functions of chassis acceleration and relative wheel movement

First we try to improve the dynamic behaviour of the chassis in the subsequent paragraphs.

The shifting of the chassis resonance (1 Hz) to lower frequencies is possible by the reduction of the spring constant. The damping may then consequently be reduced, too. These measures can improve the driving comfort. (In literature one can find the uniform opinion that the comfort relates inversely proportional to chassis acceleration).

As a very low spring constant makes technical difficulties one goes far in thinking and eliminates the spring completely. The dynamic behaviour is then only dominated by mass and damping. Box 2 shows the transfer function of the chassis suspension and its modification by the reduction of the spring constant. The root of the transfer function disappears together with the spring constant.

Shifting of the chassis resonance by $c_a \rightarrow 0$

1. Chassis

$$\frac{x_a}{x_r} = \frac{c_a + d_a p}{c_a + d_a p + m_a p^2} \text{ becomes}$$

$$\frac{x_a}{x_r} = \frac{1}{1 + \frac{m_a}{d_a} p} \text{ where } p = j\omega \text{ and } \omega = 2\pi f$$

2. Chassis and wheel (2mass oscillator)

$$\frac{x_a}{x_e} = \frac{c_a c_r + c_r d_a p}{c_a c_r + c_r d_a p + (c_a m_r + m_a (c_a + c_r)) p^2 + d_a (m_a + m_r) p^3 + m_a m_r p^4}$$

becomes

$$\frac{x_a}{x_e} = \frac{1}{1 + \frac{m_a}{d_a} p + \frac{m_a + m_r}{c_r} p^2 + \frac{m_a m_r}{c_r d_a} p^3}$$

Box 2

The spring being omitted has to be replaced by a force that carries the weight of the chassis. For this purpose an equal or constant force is to be introduced the magnitude of which is invariant to the vertical motions of the chassis (s. Fig. 4). There is no direct relation of this force to the vertical coordinate of the body or its derivative such as a spring or a damping force would have. The constant force acts as the Newtonian “reactio”.

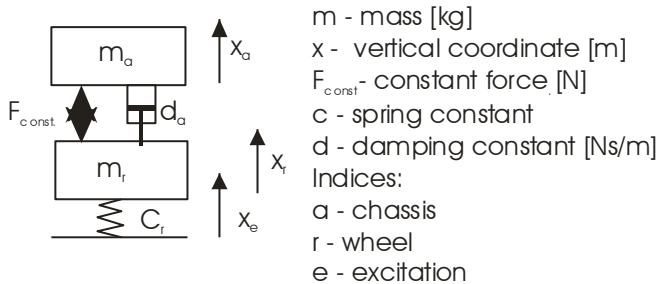


Fig. 4 Quarter car model with constant force

The new force has to be adjustable by an external signal, e.g. by an external control loop. Weight alterations and level fluctuations can so be compensated and rolling and pitching as well.

The technical implementation of the constant force shall be discussed below.

Fig. 5 shows the transfer function according to Fig.4 and Box 2. It fulfils approximately the specifications of a desired dynamic behaviour which have been discussed above.

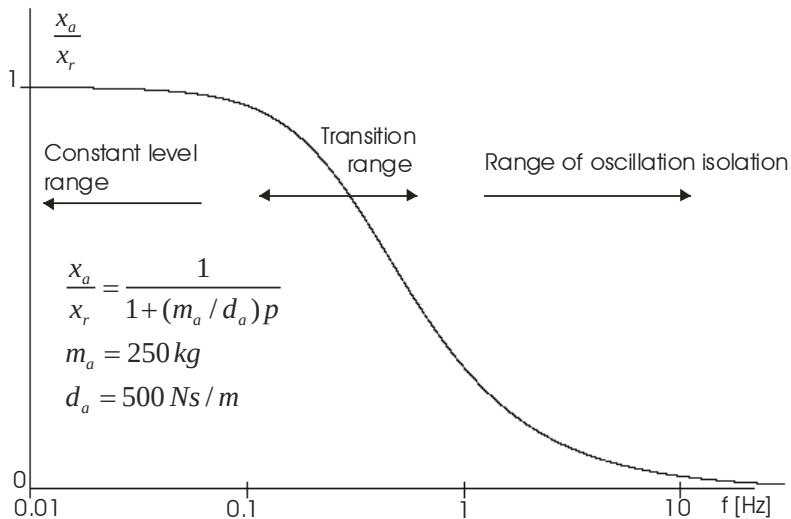


Fig. 5 Transfer function of the chassis with suppressed resonance frequency

The equations of Box 1 do not include weight forces of chassis and wheel as these forces do not contribute to the dynamics. The reset of the chassis position after having passed an obstacle will only be carried out by the dynamic quantities of the system. In the case of the reduced (suppressed) chassis resonance (omission of the spring) only the damping force remains to move the mass. The constant force has in first order no resetting task but has only to hold against the weight force. The level control function is mainly designed for the purpose of controlling system fluctuations (weight), set point modifications or transients which afford correction. The lower the

choice of the damping factor, however, the higher is the involvement of the constant force in level control.

The 10 Hz Resonance

The reduced damping constant possibly keeps no longer the amplitudes of the 10 Hz wheel resonance low enough. So we have to look for alternatives.

The 10 Hz resonance of the wheel/tyre system can be excited by road inequalities or by a wheel unbalance. A limitation of the oscillation amplitude by means of damping is always necessary mainly because of safety reasons (literature defines driving safety mostly as the inverse of the tyre's pressure fluctuations on the road).

For disturbances of a frequency >10 Hz the tyre shows a good oscillation isolation. This is, however, caused by a higher dynamic activity of the tyre. The equality of its pressure on the road gets reduced and so the driving safety (desert pistes).

The damping of the wheel/tyre system should include the 10 Hz resonance to fulfil the safety postulations. For a suspension with reduced spring and damping constant the following measures seem adequate to cope with the 10 Hz resonance. The driving comfort should desirably not be touched:

- Damping being specifically controlled by a band pass filter (approx. 10 Hz center frequency)
- Increase of the tyre damping by design

The application of a specific damping in the 10 Hz range implies no technical problems. The band pass filter gets its input from the level sensor and can be calculated by the computer of the system control (see box 3). The calculated output signal adjusts the damper. Adjustable dampers are available since the application of sky hook control.

Fig. 6 shows these relations as transfer functions. The 10 Hz resonance peak of the wheel oscillation gets limited and the vibration isolation only decreases in the controlled range of the frequency (around the mid frequency of the band pass).

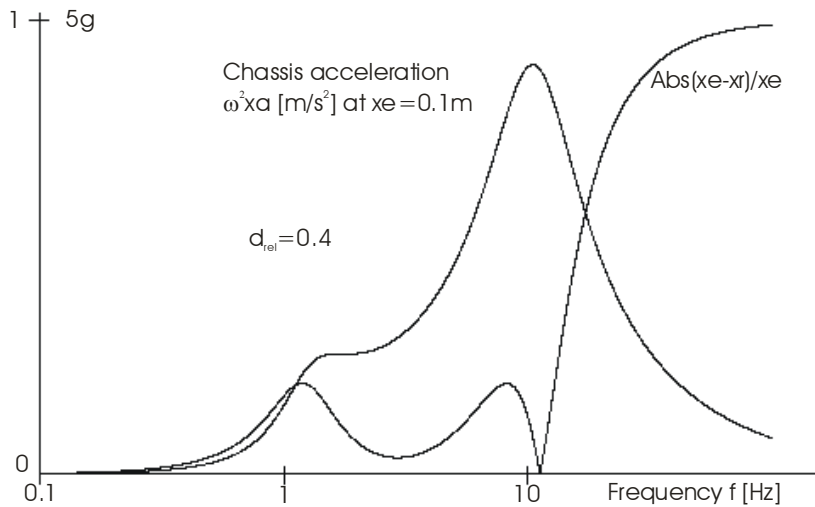


Fig. 6 Damping controlled by 10 Hz band pass filter

The total comfort which has been increased by shifting the 1 Hz resonance can be approximately kept at the same level and the driving safety might even improve as compared to a conventional system.

Band Pass Filter

$$u_e = \frac{1}{C} \int i dt + L \frac{di}{dt} + iR \text{ where } u_a = iR \text{ is}$$

$$\frac{du_a}{dt} = \frac{R}{L} u_e - \frac{1}{LC} \int u_a dt - \frac{R}{L} u_a$$

Transfer function of the band pass filter (output to input voltage) :

$$\frac{u_a}{u_e} = \frac{RCp}{1 + RCp + LCp^2} \text{ where } p = j\omega \text{ and } \omega = 2\pi f$$

center frequency : $\omega_m^2 = LC = \frac{c_r}{m_r}$

Box 3

A further possibility to build up an adequate damping for the wheel/tyre system is the application of the 2volume damping to the pneumatic tyre.

The concept needs a working volume, a damping volume and a connecting throttle. That means for the design of the tyre an outer chamber which includes the tread and an inner chamber which might be fixed to the rim and will contain some small connecting passages, the throttles (see Fig. 7). Herewith an additional damping can be achieved without increasing the rolling resistance of the tyre.

A conceivable variant would be to put the damping volume outside of the wheel, what would require a rotational connection. Eventual space problems inside the wheel might be avoided by this means.

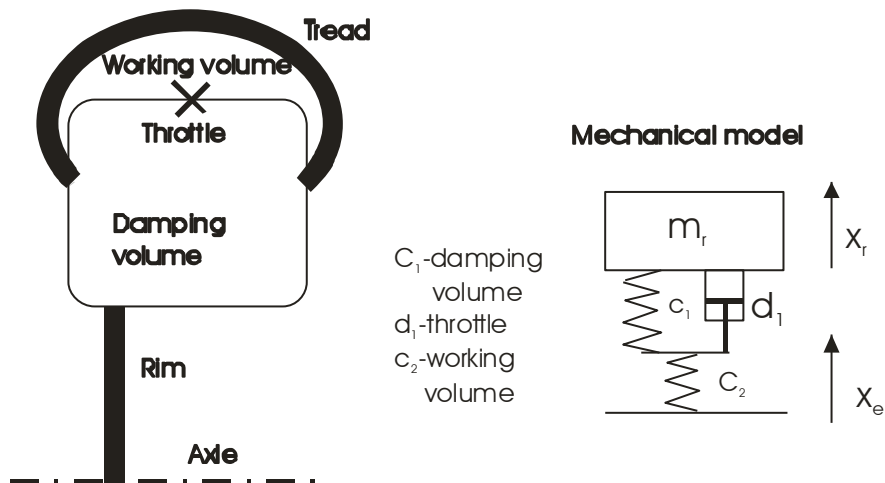


Fig. 7 Schematic of wheel and tyre with additional damping

The 2volume damping always requires a damping volume which is bigger than the working volume.

Transfer function of the 2volume damping:

$$\frac{x_r}{x_e} = \frac{c_1 + d_1 p}{c_1 + d_1 p + m_r \left(1 + \frac{c_1}{c_2} \right) p^2 + m_r \frac{d_1}{c_2} p^3} \quad \text{where } p = j\omega$$

Box 4

The magnitude of the 2volume damping is always limited. For each combination of c_1 and c_2 there is a maximum which may be adjusted by d . Fig. 8 shows the graph of the transfer function (amplitude ratio) of a tyre with a flat damping maximum.

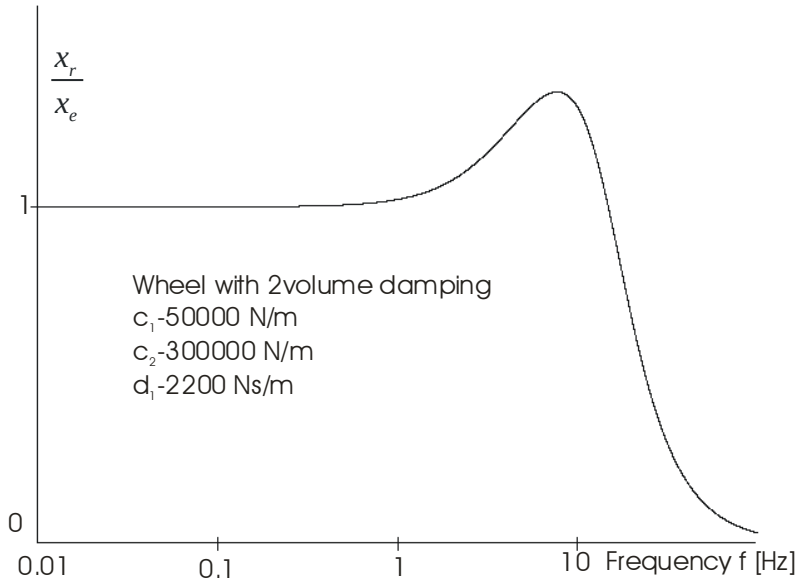


Fig. 8 Transfer function of wheel with 2volume damping

The damping of the wheel/tyre system will be supported by the residual chassis damping in the case of a total car suspension system.

Literature also deals with mass dampers to reduce wheel oscillations in the case of resonance. This method might be of interest as an additional means.

2. A Hydraulic Experiment

A Citroen XM (model 1990) got prepared for experiments. It has a hydraulic suspension system with which after small modifications experiments can be carried out in the sense of the ideas such as mentioned above. The front axle has been equipped with an experimental circuit.

The hydraulic piston associated with a wheel is connected to a pressure valve which controls its own input pressure. The oil flow is rated to a minimum and is delivered by the hydraulic supply. The oil flow (being continuous or pulsed) keeps the pressure valve on its working point (see Fig. 9).

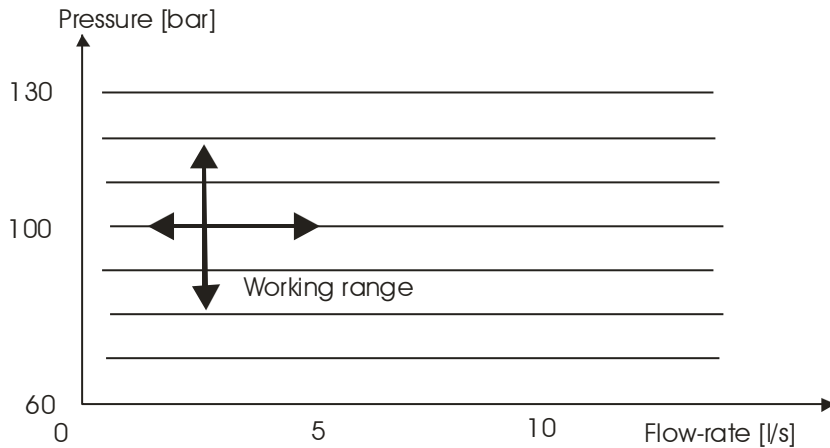


Fig. 9 Operative range of a pressure valve (magnitudes deliberate)

The level control system of the chassis consists of a distance sensor between wheel and chassis and a controller (part of the system computer) which forms the input signal for the pressure valve (see Fig. 10).

The upward movement of the wheel causes a transient increase of the flow through the pressure valve. The level adjustment (transient increase of the valve's flow resistance) leads to the replacement of the lost oil volume by the permanent minimum flow.

For improvement of the dynamics of the pressure valve a gas spring is installed upstream which buffers pressure pulses caused by rapid wheel movement. The oil flow rate will be reduced by this means.

The downward movement of the wheel is improved by the gas spring. A disadvantage is the increased oscillating potential of the wheel-/tyre system.

The non ideal dynamic properties of the pressure valve already contribute to the necessary residual damping of the chassis movements. The wanted damping can be adjusted by the installation of a further flow resistance

The adapted concept of a pressure control loop being superimposed by a level control loop presents the technical concept of the controllable equal force which has been postulated in the chapter above. The same effect could have been achieved by other means (switching valves etc.)

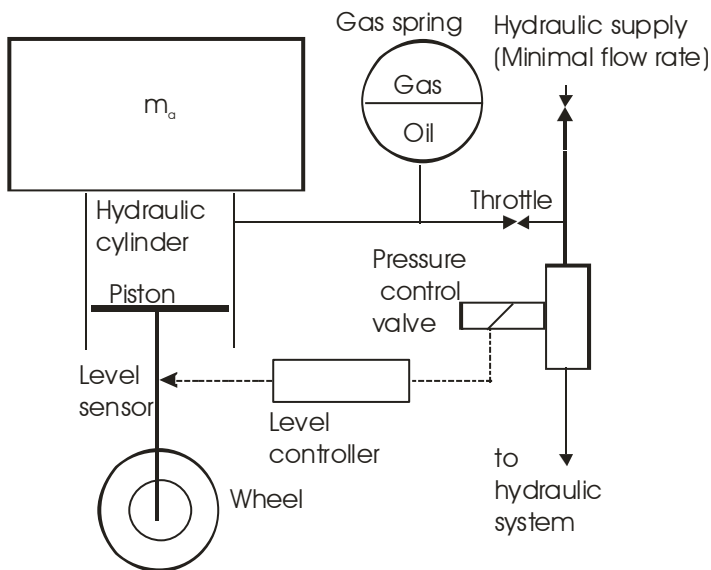


Fig. 10 Experimental hydro-pneumatic conception

The so equipped vehicle showed within the tested speed range (<60 km/h) a stable and comfortable behaviour. Additional means for the damping of the 10 Hz resonance was not necessary. The level control got a nonlinear algorithm because of stability reasons: If the control deviation exceeded a limit value the pressure in the cylinder was increase/decrease by an increment.

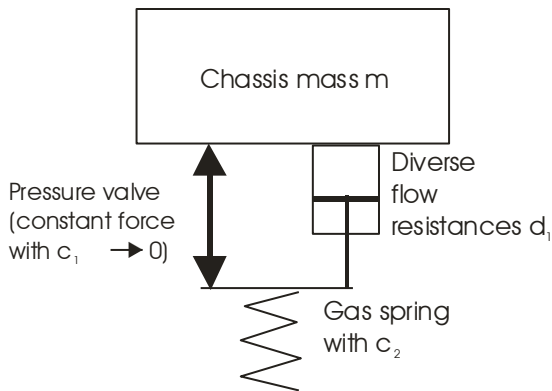


Fig. 11 Quarter car model of the hydraulic concept

The figure 11 presents the hydraulic pressure valve with level control by the parallel forces of constant power and damping. This is a simplification with respect to the pressure relation at the valve inflow in the case of dynamic transients.

The following material has been used:

Potentiometer: Firma Altmann

Pressure valve: Firma Sun with the magnetic part of Firma Fluid Team

Computer: c-control of Firma Conrad

Other components mostly stemmed from ebay

3. A Pneumatic Alternative

The hydraulic conception can be transferred to a pneumatic design without major problems. The search of an alternative development leads to a slightly different approach which shows, however, similar results. For the pneumatic design the 2volume damping shall be applied which can be found in literature frequently.

The system consists of an operative volume and a damping volume which are connected by a throttle valve (see Fig. 12). The same principle has been applied in chapter 1 for the additional damping of the pneumatic tyre.

The mechanical model of the linearised 2volume damping is a spring (operative volume) put into series with a parallel assembly of a spring and a damper (damping volume plus throttle).

There are further combinations of hydraulics and pneumatics possible.

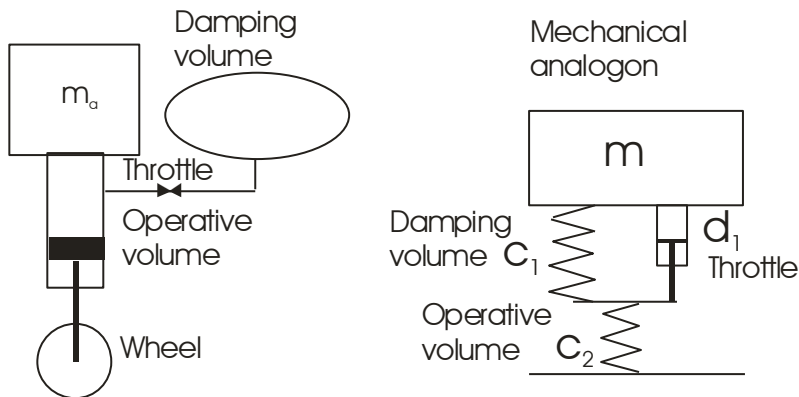


Fig. 12 2volume damping and mechanical analogon

The resonance shifting in the sense of chapter 1 can be achieved if the damping volume is large as compared to the operative volume (see Box 5). The effect of a large damping volume can be technically approximated by a pressure control loop. That avoids the disadvantages of a real large volume.

The difference to the transfer function (chassis) of box 2 is mainly caused by the spring working in series to the damping. It improves somewhat the isolation in the 10 Hz range.

The transfer function of the 2volume damping:

$$\frac{x_a}{x_r} = \frac{c_1 + d_1 p}{c_1 + d_1 p + m_a \left(1 + \frac{c_1}{c_2}\right) p^2 + m_a \frac{d_1}{c_2} p^3} \text{ where } p = j\omega$$

becomes to

$$\frac{x_a}{x_r} = \frac{1}{1 + \frac{m_a}{d_1} p + \frac{m_a}{c_2} p^2} \text{ where } c_1 \rightarrow 0$$

Box 5

The damping volume takes over the role of the constant force (equal force, $c=0$) by the means of the pressure control. The residual transfer function of the chassis is rather similar to the postulated one (see box 2 and Fig. 5).

The pressure control serves as a basis for the level control and the roll and pitch compensation.

The functional schematic of the pneumatic design is identical with the hydraulic one caused by the simplifications assumed for modelling (see Fig. 11).

It becomes apparent that the considerations of chapter 1, 2 and 3 lead to similar conceptions for the chassis suspension.

The reduced damping of the suspension concepts mentioned above may not satisfy the safety requirements for the damping of the 10 Hz resonance. As already mentioned in chapter 1 the adequate driving safety can be achieved by the following measures – being applied as single means or together:

- Band pass filter. The filter output signal has a limited bandwidth with a maximum at 10 Hz adjusts the adherent damper.
- Compensating masses may be useful as additional means

- Additional damping of the pneumatic tyre. The application of the 2volume principle allows for an increased damping constant without increasing the rolling resistance.

Fig. 13 shows the transfer function of a 2mass oscillator which has 2volume dampings for chassis and wheel (parameters see box 6).

The wheel/tyre system contributes to comfort and safety. At perturbations > 10 Hz the ground adhesion decreases and lane keeping gets more problematic (desert pistes).

Below of the 10 Hz resonance the adhesion force of the tyre is better equalised but the chassis suspension has a higher contribution to the driving comfort.

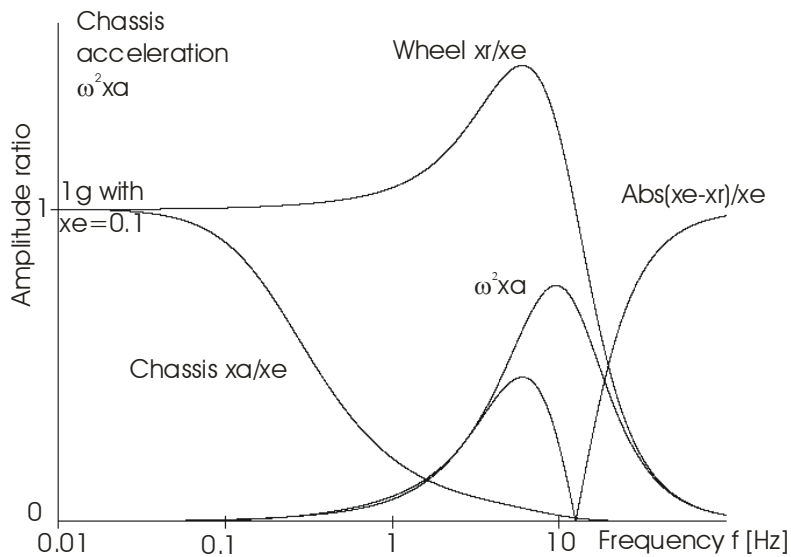


Fig. 13 2volume damping for chassis and wheel (transfer functions)

The positive contributions of the tyre spring to the behaviour of the suspension are to see in the improved oscillation isolation at higher frequencies and a good adhesion at lower frequencies. In praxis the road always delivers a mixture on which the wheel/tyre systems reacts in a favourable way.

4. Step Response

The behaviour of the suspension system can also be demonstrated by step response. The relation to the transfer function is given by the frequency content of the step function. At the beginning there are high frequencies and with time the frequency decreases to zero.

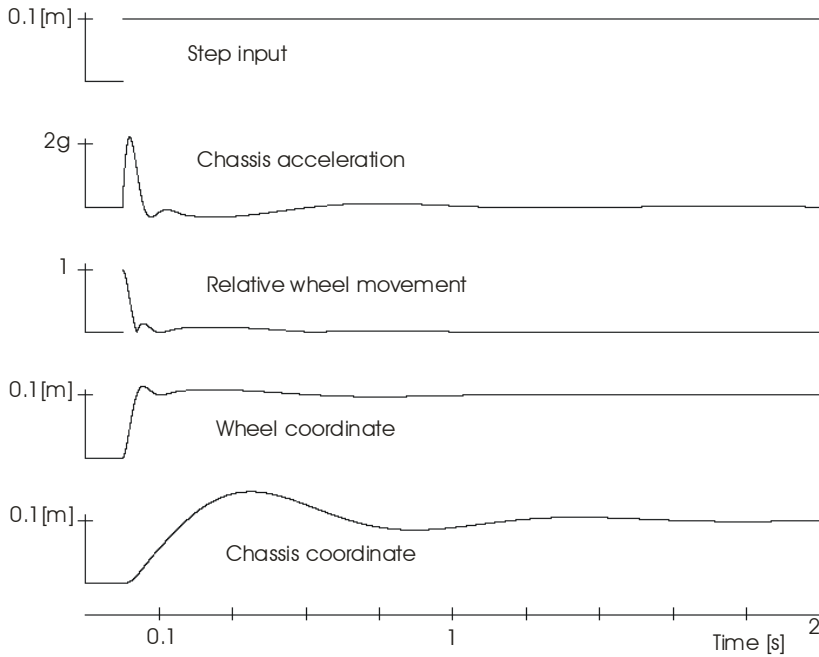


Fig. 14 Step response of the conventional system (model see Fig. 1 + parameters see Fig. 2)

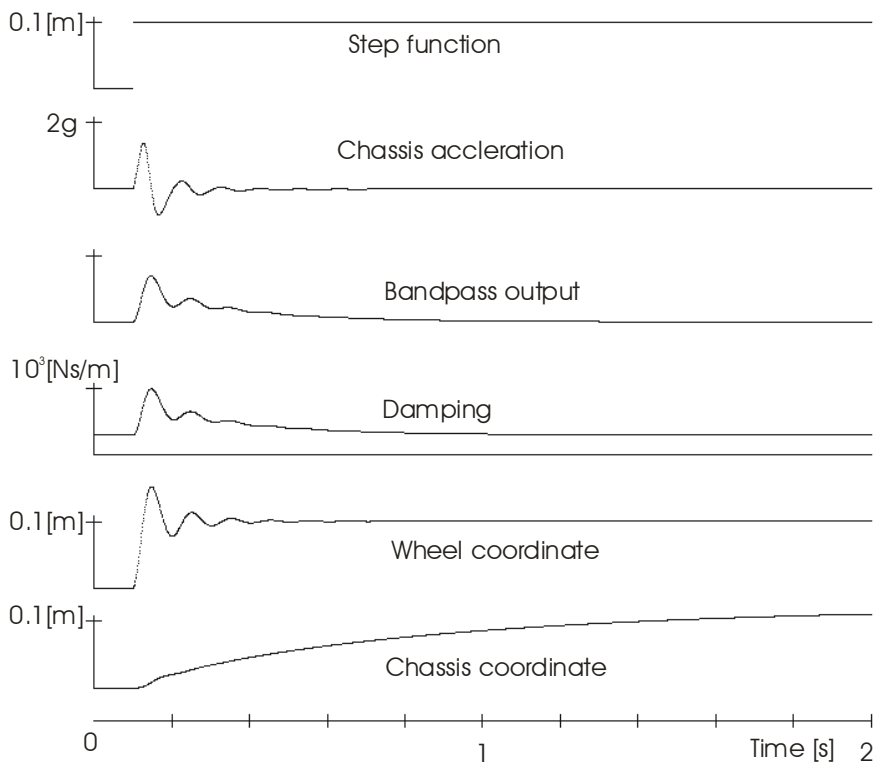


Fig. 15 Step response with controlled damping (band pass)

The suspension system of Fig. 15 consists of a chassis suspension with a reduced resonance frequency following chapter 1, the damping of which is controlled by a band pass filter with the mid frequency of about 10 Hz. The optimal adjustment of the filter contains still large margins, where the introduction of time constants seems advantageous.

In Fig. 16 a system is presented which has 2volume damping for wheel and chassis. It is evident that the active volume in the chassis arrangement isolates the oscillations with higher frequencies. The damping of the wheel/tyre system just limits its inherent capacity of oscillation.

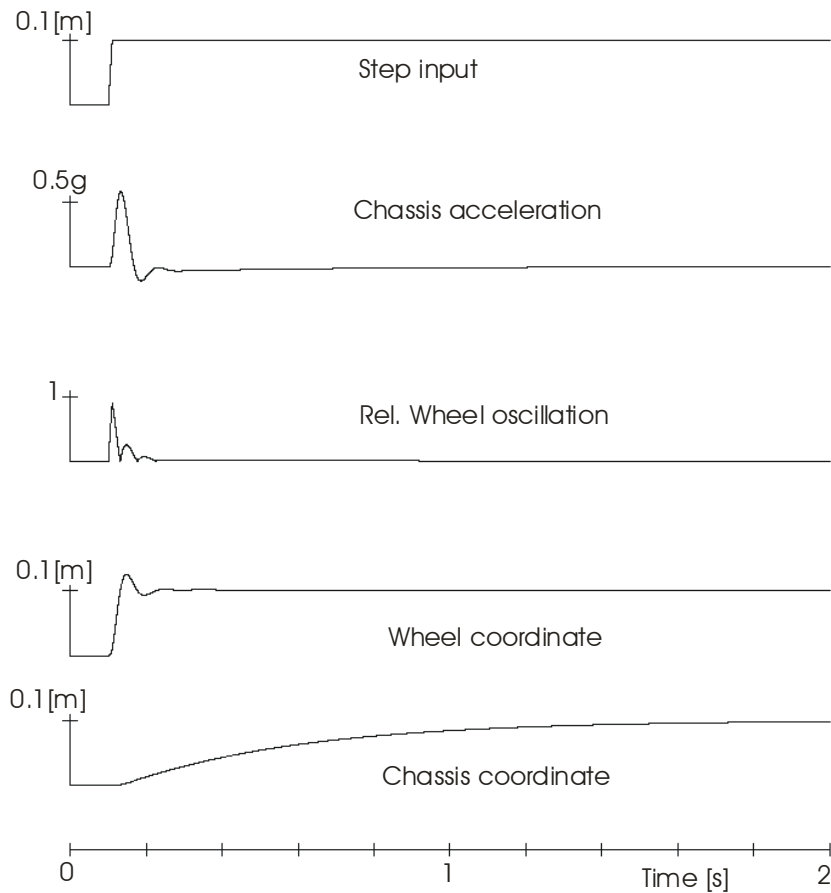


Fig. 16 Step response with 2volume damping for chassis and wheel

5. Energy Aspects

A mass of 1 ton requires 1 KW for vertical moves of 0.1 m/sec. The spring of the car suspension which is located between wheel and chassis generally serves to equalise such motions by buffering the energy.

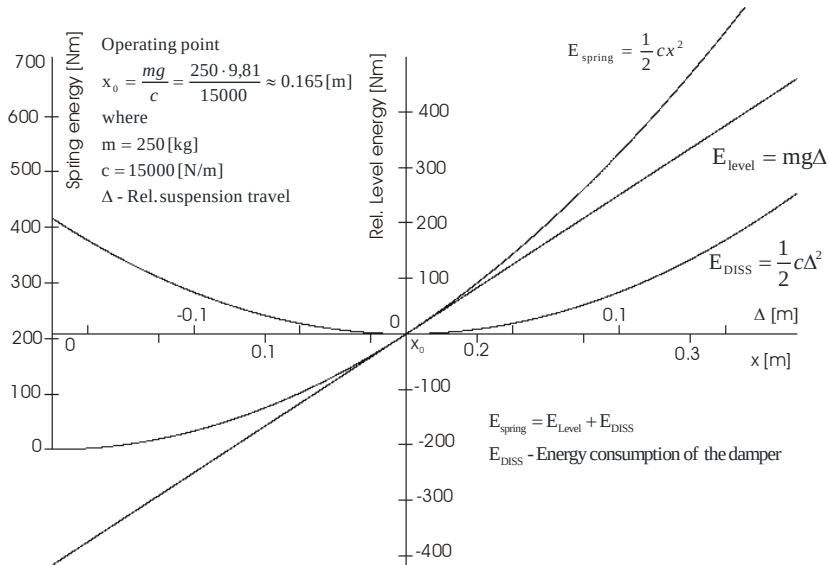


Abb. 17 Energies at chassis and spring

Operation point x_0 :

$$x_0 c = mg \rightarrow x_0 = \frac{mg}{c} \quad (\text{spring force} = \text{weight})$$

spring energy :

$$E = \frac{1}{2} c x^2$$

with $x = x_0 + \Delta$ is

$$E = \frac{1}{2} c (x_0 + \Delta)^2 = \frac{1}{2} c x_0^2 + c x_0 \Delta + \frac{1}{2} c \Delta^2$$

The energy difference caused by Δ is :

$$\delta E = c x_0 \Delta + \frac{1}{2} c \Delta^2 = mg \Delta + \frac{1}{2} c \Delta^2$$

$mg \Delta$ - is needed for level compensation

$\frac{1}{2} c \Delta^2$ - is dissipated as heat

Box 7

As the spring travel stores more energy than the following level compensation of the chassis needs a surplus of energy exists which causes oscillations. The damper converts the mechanical energy into heat and dissipates it to ambient (see Fig. 17 + Box 7).

The energy dissipation of the damper would amount to approx. < 500 Watts in the example above.

If there is no energy buffer the energy for level compensation has to be supplied by an active force. The dissipation via damper would not exist in this case.

The hydro pneumatic experiment did not need much energy as could be estimated by the temperature of the pressure control valves and the switching frequency of the car's hydraulic pressure controller.

There are several methods thinkable to keep the energy consumption of an active suspension.

Pitching and Rolling

Lateral and longitudinal forces at the chassis cause these phenomena. Their level compensation is in the systems described above task of the individual wheel/chassis level control. In addition there is place for many measures to improve the dynamics.

The appropriate sensors to initiate countermeasures could probably be horizontal chassis acceleration, motion of steering and/or brake pedal.

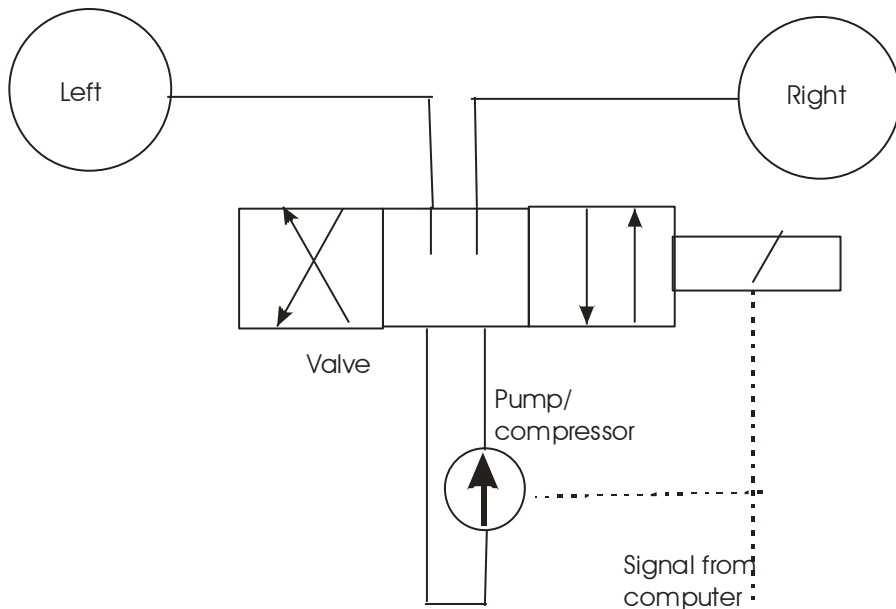


Fig. 18 Support of the roll compensation

Fig. 18 shows the example of a pumping system which may help to compensate movements of the chassis caused by lateral forces.

7. Preview

A speed racer in alpine ski competitions has similar problems as a racing car designer. The air resistance has to be optimised and there has to be sufficient “spring travel” at disposition to cope with average impedances.

The alpine racer has his leg muscles in the mode “equal force (constant force)” adjusted to hold his body weight and to equalise sudden inequalities of the piste. So he can overcome without preview even unexpected small obstacles shock free.

Only when the obstacles exceed the “spring travel” at disposition preview with adherent measures is desirable and necessary. At challenging manoeuvres in difficult terrain the precision of preview decide on the success of manoeuvres.

Fig. 19-21 show an engaged racer at some manoeuvres

The speed racer has limited possibilities to act in the vertical. He can certainly adapt the spring travel according to the expected obstacle size. So he can keep his body free from vertical acceleration and can stabilise his direction by applying a pressure pulse on his skis..

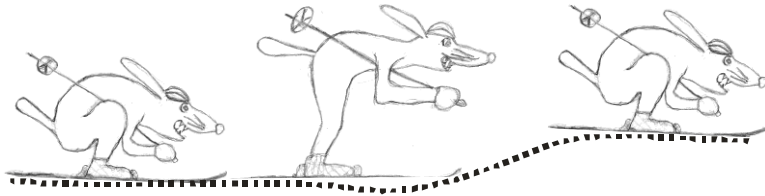


Fig. 19 Disposal of “spring travel”

Working with inertia leads to the skiing classics high relief and deep relief. All skiers know these exercises. They mostly get applied to initiate a change of direction . The speed racer also uses them to get over an obstacle or to limit the jump over a hill.

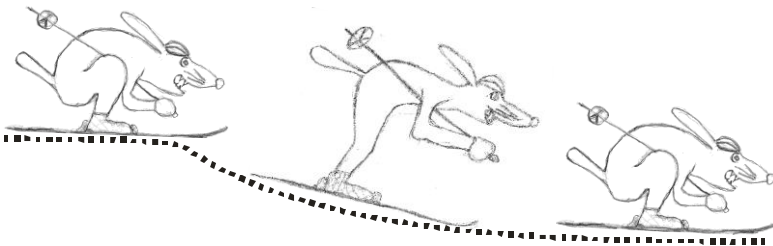


Fig. 20 Controlling an inequality

Alternatively the racer may apply deep relief and only reduce his muscle power when gliding over an obstacle. He then saves energy and runs smoother. He, however, comes out of the procedure with a lower mass center and a lower bottom pressure of the skis.

The ski racer may never reduce his bottom pressure when he feels that his body is under the influence of an angular momentum because he may lose the direction. Spectacular falls may develop from an inherent angular momentum if bottom pressure is lost.

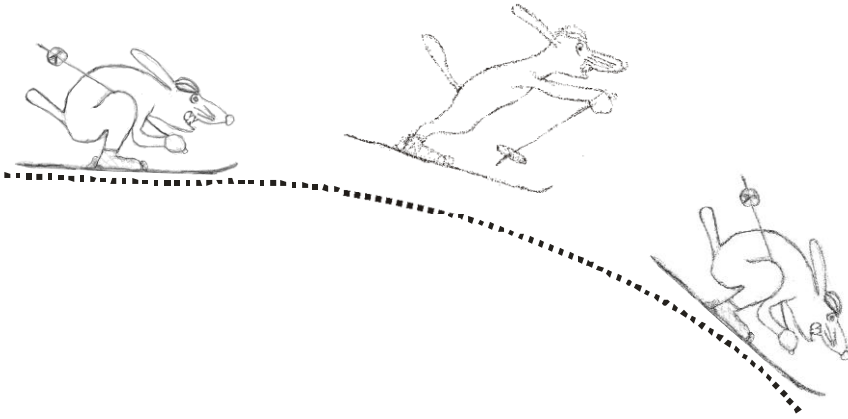


Fig. 21 Active jumping of a slope

A car suspension having remote control of the vertical forces can always be used in conjunction with preview. The development of proper optical sensors might improve vertical dynamics.

Annex 1

Linear pneumatic spring

Ideal gas law

$$P \cdot V = M \cdot R \cdot T$$

Logarithmic notation

$$\ln P + \ln V = \ln M + \ln R + \ln T$$

Differential notation

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dM}{M} + \frac{dR}{R} + \frac{dT}{T}$$

With $dM=dR=dT=0$ is

$$dP = -\frac{P}{V} dV$$

With $dP=dF/A$ und $dV=-A dx$ is

$$dF = \frac{P}{V} \cdot A^2 \cdot dx$$

For the case of $dT>0$ is

$$dF = n \cdot \frac{P}{V} \cdot A^2 \cdot dx \quad \text{With } 1 < n < 1.4$$

The term $n \cdot \frac{P}{V} \cdot A^2$ Corresponds to the mechanical spring constant c.

P - pressure, V -Volume, M - mass, R - gas constant, T - absolute temperature,
F -force, A - area, x - local coordinate; d - signifies small changes

Annex 2) Car suspension with 2volume damping (pneumatic)

Nomenclature as annex 1)

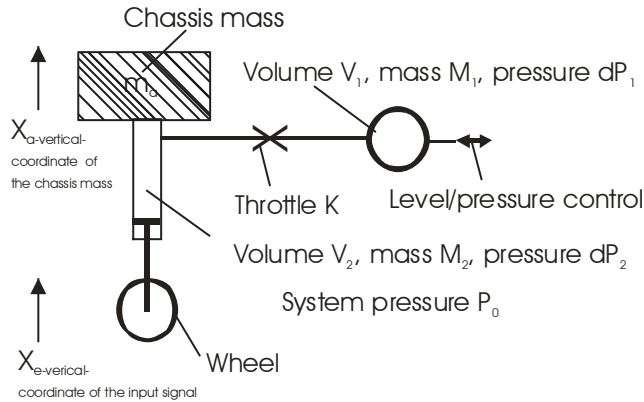


Fig. 1) System schematic

Equations (ideal gas law)

$$P \cdot V = M \cdot R \cdot T$$

$$\ln P + \ln V = \ln M + \ln R + \ln T$$

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dM}{M} + \frac{dR}{R} + \frac{dT}{T}$$

mit $dR = 0$ und $dT = 0$ ist

$$dP = -\frac{P}{V} dV + \frac{P}{M} dM$$

Application

Pressure variations at V_1 and V_2 (linearised):

$$dP_2 = -\frac{P_0}{V_2} dV_2 + \frac{P_0}{M_2} dM_2$$

$$dP_1 = \frac{P_0}{M_1} dM_1$$

Flow through K linearised)

$$d\dot{M}_1 = K \cdot (dP_2 - dP_1)$$

$$dM_2 = -dM_1$$

$$d\dot{M}_1 = K \cdot dP_2 - K \frac{P_0}{M_1} dM_1$$

$$dP_2 = \frac{1}{K} d\dot{M}_1 + \frac{P_0}{M_1} dM_1$$

Transformation to Laplace with d/dt to p=jw

$$dP_2 = \frac{p}{K} dM_1 + \frac{P_0}{M_1} dM_1$$

$$dP_2 = \left(\frac{p}{K} + \frac{P_0}{M_1} \right) \cdot dM_1$$

$$dM_2 = -dM_1 = dP_2 \frac{1}{\frac{p}{K} + \frac{P_0}{M_1}}$$

$$dP_2 = -\frac{P_0}{V_2} dV_2 - \frac{P_0}{M_2} \frac{1}{\frac{p}{K} + \frac{P_0}{M_1}} dP_2$$

$$dP_2 \left(1 + \frac{P_0}{M_2} \frac{1}{\frac{p}{K} + \frac{P_0}{M_1}} \right) = -\frac{P_0}{V_2} dV_2$$

$$dP_2 \left(\frac{p}{K} + \frac{P_0}{M_1} + \frac{P_0}{M_2} \right) = - \left(\frac{P_0 p}{V_2 K} + \frac{P_0 P_0}{V_2 M_1} \right) dV_2$$

$$dP_2 = \frac{p / K + P_0 / M_1}{p / K + P_0 / M_1 + P_0 / M_2} \frac{P_0}{V_2} dV_2$$

$$dF_2 = A_2 \cdot dP_2$$

$$dV_2 = -A_2 (x_e - x_a)$$

$$dF_2 = \frac{A_2^2 P_0}{V_2} \frac{p + K P_0 / M_1}{p + K P_0 (1 / M_1 + 1 / M_2)} (x_e - x_a)$$

Second law of .Newton :

$$m_a x_a p^2 = dF_2$$

$$m_a x_a p^2 = \frac{A_2^2 P_0}{V_2} \frac{p + KP_0 / M_1}{p + KP_0 (1/M_1 + 1/M_2)} (x_e - x_a)$$

$$x_a \cdot \left(m_a p^2 + \frac{p + KP_0 / M_1}{p + KP_0 (1/M_1 + 1/M_2)} \frac{A_2^2 P_0}{V_2} \right)$$

$$= \left(\frac{p + KP_0 / M_1}{p + KP_0 (1/M_1 + 1/M_2)} \frac{A_2^2 P_0}{V_2} \right) \cdot x_e$$

$$x_a \cdot \left(m_a p^3 + \frac{m_a KP_0}{1/M_2 + 1/M_1} p^2 + \frac{P_0 A_2^2}{V_2} p + \frac{KP_0^2 A_2^2}{V_2 M_1} \right)$$

$$= x_e \left(\frac{P_0 A_2^2}{V_2} p + \frac{KP_0 A_2^2}{V_2} \right)$$

Transfer function :

$$\frac{x_a(p)}{x_e(p)} = \frac{1 + \frac{M_1}{KP_0} p}{1 + \frac{M_1}{KP_0} p + \frac{m_a V_2 M_1}{A_2^2 P_0} (1/M_1 + 1/M_2) p^2 + \frac{m_a V_2 M_1}{KA_2^2 P_0^2} p^3}$$

The simplified transfer function for big volumes V_1 can be derived by putting M_1 to infinite:

$$\frac{x_a(p)}{x_e(p)} = \frac{1}{1 + \frac{m_a V_2 K}{A_2^2} p + \frac{m_a V_2}{A_2^2 P_0} p^2}$$

In the present application the big volume shall be simulated by a pressure control installation. The average pressure which is determined by chassis weight and the actual pressure are compared. The difference is turned to zero by adding or subtracting air. In parallel the level control works quasi stationery.

A mechanical analogon:

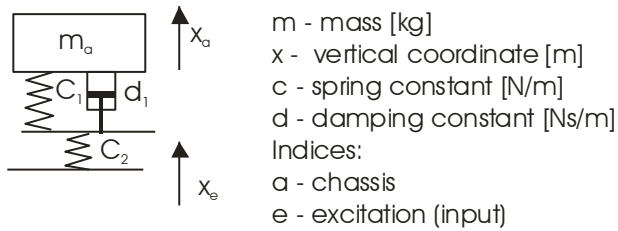


Fig. 2) Analogon with spring and damper

Transfer function:

$$\frac{x_a(p)}{x_e(p)} = \frac{1 + \frac{d_1}{c_1} p}{1 + \frac{d_1}{c_1} p + m_a \left(\frac{1}{c_1} + \frac{1}{c_2} \right) p^2 + \frac{m_a d_1}{c_2} p^3}$$

Simplification with c_1 to 0:

$$\frac{x_a(p)}{x_e(p)} = \frac{1}{1 + \frac{m_a}{d_1} p + \frac{m_a}{c_2} p^2}$$

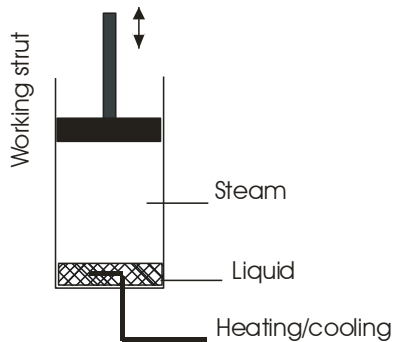
The correspondence to the pneumatic 2volume damping is evident.

Annex 3

Equal force concept on thermal basis (water)

Annex 3)

Equal force concept on thermal basis (water)



Formula approximating steam pressure=f(liquid temperatur):

$$p[bar] = \left(\frac{T[^\circ C]}{100} \right)^4$$

180°C correspond to approx. 10.5 bar. The energy to vaporise 1kg Wasser amounts to approx. 2088 KJ, the expansion energy is approx. 169 KJ. Linearisierung im Arbeitspunkt T=180°C:

$$\frac{dp}{d(T/100)} = 4 \left(\frac{T}{100} \right)^3$$

$$dp = \frac{4 \cdot 5.832}{100} dT = 0.2333 \cdot dT$$

Etc., to be continued...

Annex 4) Level control (linear)

System configuration see Annex 2) Fig. 1

The level control (here set point=0) will be carried out by adjusting the pressure set point of the damping volume (dP_1 of V_1). This input acts over-riding to the compensation of inequalities and is mainly efficient at lower frequencies.

$$dP_{setpoint} = P - factor \cdot (x_e - x_a) + I - factor \cdot \int (x_e - x_a) dt$$

The in/out flow to V_1 amounts to:

$$d\dot{M}_3 = throttle\ factor \cdot (dP_1 - dP_{setpoint})$$

The air flows into the 2volume system from the air supply via a controlled valve and out via a compressor to the air supply or via valve to a low pressure vessel.

Pitching and rolling control can be improved by adding proper sensors such as horizontal acceleration.

Computer programme for an estimate of the dynamics:

ma = 250	'chassis mass [kg]
v1 = 0.0001	'damping volumen [m^3]
v2 = 0.0005	'operative volumen [m^3]
p0 = 1000000	'operative pressure [N/m^2]
a0 = 2500 / p0	'piston area [m^2]
m1 = 0.00013	'gas mass in the damping volume [kg]
k1 = 0.00000005	'throttle constant between the volumes [(kg/sec)/(N/m^2)]

```

k3 = 0.0000005      'throttle constant to supply [(kg/sec)/(N/m^2)]

m2 = 0.00065        'gas mass in the operative volume [kg]

dt = 0.0001         'time increment [sec]

marke01:

t = t + dt          'running time [sec]

If t > 1 Then xe = 0.1  'step input [m]

If t > 3 Then f = 1000  'step function of a force

If t > 10 Then f = 0    'e.g. vert. Component of a.centrifugal force[N]


aa = (1 / ma) * dp2 * a0 - (1 / ma) * f 'acceleration of the chassis [m/sec^2]

va = va + aa * dt      'velocity of the chassis [m/sec]

xa = xa + va * dt      'local coordinate [m]


xai = xai + xa * dt     'time intergral of chassis cordinate [msec]

xei = xei + xe * dt     'time integralof the road ckordinate [msec]


dp2 = (p0 / v2) * a0 * (xe - xa) - (p0 / m2) * dm1 'pressure in the operative
volume [N/m^2]

dp1 = (p0 / m1) * dm1 - (p0 / m1) * (dm3)          'pressure in the damping
volume[N/m^2]

dm1p = (dp2 - dp1) * k1                            'mass flow between the Vol-
umes[kg/sec]

dm1 = dm1 + dm1p * dt

```

$dm3p = (dp1 - psoll) * k3$ 'mass flow of the level control
[kg/sec]

$dm3 = dm3 + dm3p * dt$

$xstern = (xstern + 0.001 * xa) / 1.001$

$pstern = (pstern + 0.0001 * dp1) / 1.0001$

$fstern = (fstern + 0.0002 * f) / 1.0002$

$psoll = 1000 * (xe - xa) + 1000 * (xe - xa) * f_$

$+ 1000 * (xei - xai) + 400 * f + 15 * (f - fstern)$ 'pressure set point of the
level control

$PSet(1000 + 1000 * t, 2000 - 10000 * xe)$

$PSet(1000 + 1000 * t, 2000 - 10000 * xa)$

$PSet(1000 + 1000 * t, 5000 - 0.0025 * dp2)$

$PSet(1000 + 1000 * t, 3000 - 1 * f)$

$PSet(1000 + 1000 * t, 8000 - 1000000 * dm1)$

$PSet(1000 + 1000 * t, 9000 - 0.0025 * psoll)$

$PSet(1000 + 1000 * t, 7000 - 50000 * dm3p)$

$PSet(1000 + 1000 * t, 13000 - 0.0025 * dp1)$

$PSet(1000 + 1000 * t, 11000 - 1 * (f - fstern))$

$PSet(1000 + 1000 * t, 13000 - 200 * aa)$

If $t < 15$ Then GoTo marke01

End Sub

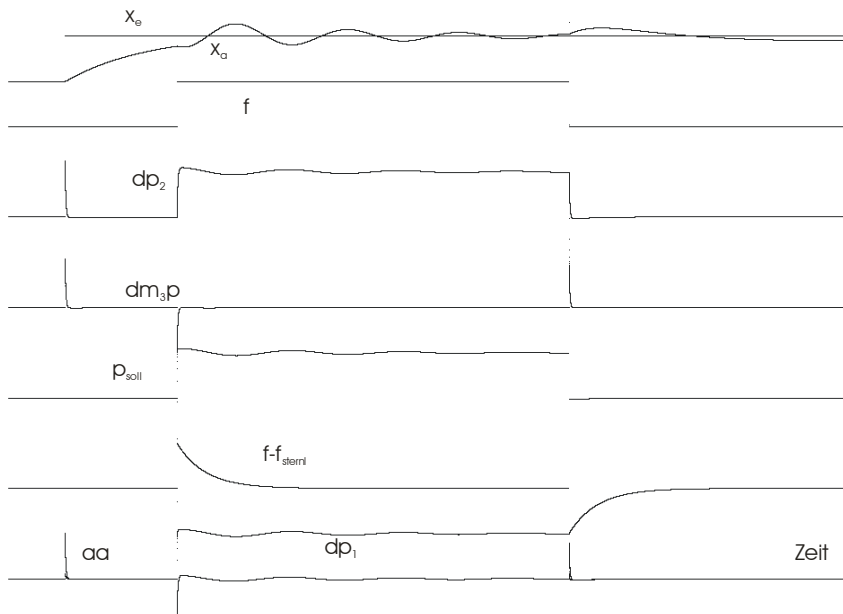


Fig.1) Step inputs of road inequalities and centrifugal force